

# Multi-circuit system observational data fusion method

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**Abstract.** At present, the fusion method of multi-circuit system observation data is complicated and the calculation process is large, which is difficult to be extended to the observation application of grid space. This paper presents a high-precision multi-circuit system observation data fusion method. Firstly, a high precision multi-circuit system observational data fusion method is put forward on the basis of the redundancy set estimation (hereinafter referred to as RSE for short), which combines the process of the information fusion with the set operation link of the algorithm itself. Finally, in order to verify the feasibility and effectiveness of the method studied in this paper, the grid observational data fusion simulation experiment is carried out. This method can guarantee the relatively high observation precision, and at the same time, does not significantly increase the amount of calculation of the single-circuit system redundancy set estimation algorithm, therefore, is of relatively high real-time performance.

**Key words.** Multi-circuit system, active observational data fusion, redundancy set estimation, optimal angle.

## 1. Introduction

In the current society, the application of the multi-circuit systems is becoming more and more popular, and various multi-circuit systems with high intelligence are playing or about to play an important role in the different occasions. However, with the continuous extension of the fields of the human social activities and the continuous development of the circuit systems research, the single multi-circuit system is faced a number of difficulties in the replacement of human beings to accomplish the missions including large-scale disaster relief, scientific inspection and so on as well as the battlefield environmental monitoring and other military missions, for example: poor reliability, small operating range, and low task accomplishment efficiency, etc. While the relevant researches show that, the multi-circuit system composed of a number of circuit systems can exactly solve these problems through the coordination and cooperation [1, 2]. Therefore, the multi-circuit system is considered to be of broad application prospects.

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At present, majority of the passive data fusion methods are based on certain estimation method, and the most widely used method is the traditional estimation method on the basis of the probabilistic statistical theory. Among them, literature [3] made use of the Kalman filter method to deduce the relationship between the covariance matrix and the state variables of the multi-circuit system, and it has proven that the covariance matrix of the minimized target position is equivalent to the maximum eigenvalue of the minimized target covariance matrix, so as to obtain the condition of optimal observation of the multi-circuit system through calculation. In literature [4], the problem of observing ground moving targets through Doppler-based adaptive observer was studied. In this paper, the relationship between the observation results and the noise as well as the observation angle of the single circuit system was discussed. And then the Fisher information matrix and Cramer-Rao boundary principle was adopted, by optimizing the covariance matrix of the observation results, the conditions of optimal observation were obtained, so as to realize the optimization of the position and velocity of the ground targets. In literature [5], the authors assumed that the target states observational by several different circuit systems obeyed the normal distribution, and then applied the theory of distribution multiplication to fuse the multiple observations so as to obtain a more precise observational data fusion with smaller uncertainty, and achieve the purpose of the multi-circuit system coordinated observation to improve the measurement precision. In view of this, a new nonlinear filtering method, namely, the Extended Set-membership Filter (hereinafter referred to as RSE for short), has been proposed and applied to the fusion of observational data. This method only requires the noise distribution to be bounded, which can be satisfied during the actual observation [6]. In literature [7], the authors tried to solve the problem of multi-circuit system observational data fusion by the application of the RSE method.

In this paper, a method of active observational data fusion of the multi-circuit system on the basis of RSE algorithm in the grid space is proposed. This method includes two parts: Firstly, on the basis of the detailed analysis of the work in literature [8], the algorithm is improved, and the observational data fusion algorithm of the multi-circuit system and the RSE estimation algorithm are merged organically to improve the real-timing, and precision of the data fusion algorithm. Then, by the optimization of the observation conditions, a method of coordinated planning of the behavior of the multi-circuit systems by the application of the concept of optimal angle is put forward to optimize the observation results, so that the active observational data fusion of the multi-circuit system can be finally realized. Therefore, in this paper, both the circuit system and the moving target are represented by the following kinematic model on the basis of Newton's law of motion (after discretization), respectively expressed as the following:

$$\begin{cases} p_{i,k+1}^R = p_{i,k}^R + v_{i,k}^R \cdot \Delta T, \\ v_{i,k+1}^R = v_{i,k}^R + a_{i,k}^R \cdot \Delta T, \\ a_{i,k+1}^R = u_{i,k}, \end{cases} \quad (1)$$

$$\Gamma(p_{i,k}^R, v_{i,k}^R, a_{i,k}^R) \leq 0,$$

$$\begin{cases} p_{k+1}^T = p_k^T + v_k^T \cdot \Delta T + \omega_{1,K}, \\ v_{k+1}^T = \omega_{2,k}, \end{cases} \quad (2)$$

$$y_{i,k} = h_i(p_k, v_k) + \omega_{3,i,K}.$$

Here, the superscripts  $R$  and  $T$ , respectively, stand for the circuit system and the target system;  $p_{i,k} = (x_{i,k}, y_{i,k}, z_{i,k})^T$  ( $i = 1, 2, \dots, n$ ) stands for the position of the system at the time  $k$  of the  $i$ -th circuit system;  $v_{i,k}^R = (v_{i,x,k}, v_{i,y,k}, v_{i,z,k})^T$  and  $a_{i,k}^R = (a_{i,x,k}, a_{i,y,k}, a_{i,z,k})^T$  stand for the velocity and the acceleration at the time  $k$  respectively;  $u_{i,k}$  stands for the control input of the  $i$ -th circuit system at the time  $k$ ; it is assumed in this paper that the velocity variation of the circuit system  $u_k$ , that is, the acceleration is the controllable input of the circuit system; the sampling time; the motion constraint inequation of the system of the circuit system;  $\Delta T$  stands for the sampling time,  $\Gamma(\cdot, \cdot, \cdot)$  stands for the motion constraint inequation of the circuit system;  $p_k^T$  stands for the position information of the target circuit at the time  $k$ , the corresponding  $v_k^T$  stands for the velocity information of the target circuit at the time  $k$ ;  $y_{i,k}$  stands for the observation value of the target circuit by the  $i$ -th circuit system at the time  $k$ . In this paper, it is assumed that the circuit system does not have any priori knowledge of the motion performance of the target circuit, and thus it is impossible to conduct accurate prediction. Therefore, only the second-order motion equation is considered, rather than the third-order equation of motion as shown in the circuit system (1). Symbols  $\omega_{j,k}$  ( $j = 1, 2$ ) stand for the process noise of the moving object;  $\omega_{3,i,k}$  stands for the measurement noise when the target circuit is observed by the  $i$ -th circuit system. According to the assumption, all the three noise vectors shall satisfy the following conditions:

$$\omega^T Q^{-1} \omega \leq 1.$$

where  $Q$  is a positive definite symmetric matrix.

In this paper, it is assumed that the observation of all the circuit systems on the target is implemented by the adaptive observer, and the observation equations are

$$r_{i,k} = \left[ (x_{T,k} - x_{i,k})^2 + (y_{T,k} - y_{i,k})^2 + (z_{T,k} - z_{i,k})^2 \right]^{1/2} + n_{r,i}, \quad (3)$$

$$\theta_{i,k} = \tan^{-1} \frac{z_{T,k} - z_{i,k}}{\left[ (x_{T,k} - x_{i,k})^2 + (y_{T,k} - y_{i,k})^2 \right]^{1/2}} + n_{\theta,i}, \quad (4)$$

$$\alpha_{i,k} = \tan^{-1} \frac{y_{T,k} - y_{i,k}}{x_{T,k} - x_{i,k}} + n_{\alpha,i}. \quad (5)$$

In these three equations,  $r_{i,k}, \theta_{i,k}, \alpha_{i,k}$  stand for the three observations obtained by the adaptive observer, as shown in Fig. 1. There,  $(n_{r,i}, n_{\theta,i}, n_{\alpha,i})^T$  stand for the

measurement of the noise, namely, the  $\omega_{3,i,k}$  in equation (2). It should be noted that if the observer is not an adaptive observer, the corresponding observation equation  $h(\cdot)$  will be changed accordingly, but this will not affect the application of the observational data fusion method described in this paper. In addition, different circuit systems can also carry different observers, and the corresponding observation equation will be different as well.

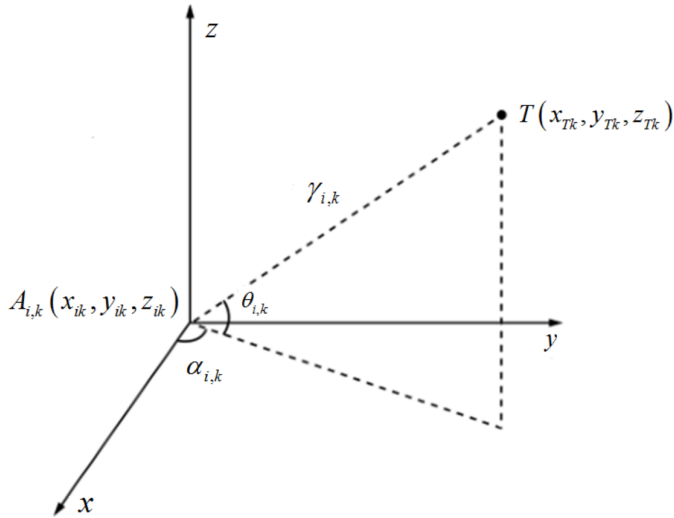


Fig. 1

## 2. Observational data fusion method on the basis of RSE

This section mainly solves how to conduct fusion on the Observational data for the multi-circuit system in the active Observational data fusion, so as to obtain more precise target state information, that is, the passive Observational data fusion problem.

According to the principle of the RSE method, when observing the target with a circuit system, the target should be included in the observational observation set; if another circuit system observes the same target simultaneously, another observation set containing the target shall be obtained, and the rest can be deduced in the same manner, the target should be included in the intersection of the observations set and the predictions set of these circuit systems. Figure 2 shows the diagram of the observational data fusion of two circuit systems. Therefore, by solving the intersection of these sets more accurate results can be obtained.

In this paper, the fusion method of observational data proposed makes use of the characteristics of RSE's own calculation, and integrates this intersection solving process into the estimation method, that is, the process of solving the intersection of the prediction set and the observation set is replaced by solving the intersection of the prediction set and two observation sets. The detailed procedure of the algorithm

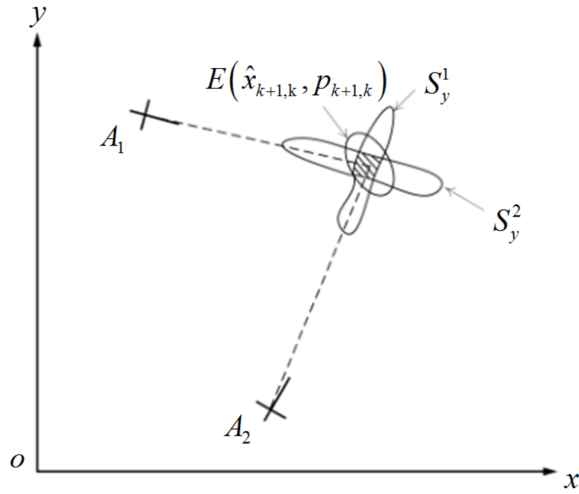


Fig. 2. Basic idea of the observational data fusion method

is shown as the following (the variable with the left superscript sign  $i$  ( $i = 1, 2, \dots, n$ ) indicates that the variable is calculated from the observational data of the first circuit system. For the ease of understanding, this paper only sets out the algorithm with one collaborative circuit system, and the multiple collaborative circuit system method can be deduced in the same manner):

Step 1. Initialization

$$\hat{x}_{k,k} = \hat{x}_I, P_{k,k} = P_I.$$

Step 2. Single machine forecasting

$$\hat{x}_{k+1,k} = f(\hat{x}_{k,k}), \tag{6}$$

$$P_{k+1,k} = \frac{A_k P_k A_k^T}{1 - \beta_k} + \frac{\hat{Q}_k}{\beta_k}. \tag{7}$$

Here,

$$A_k = \frac{\partial f(x_k)}{\partial x} \Big|_{x_k = \hat{x}_{k+1,k}}.$$

Step 3. Collaborative updates

Step 3.1. Fusion of the collaborative circuit system observational data

$${}^2\hat{x}_{k+1,k+1} = \hat{x}_{k+1,k} + {}^2K_k ({}^2y_{k+1} - h(\hat{x}_{k+1,k})), \tag{8}$$

$${}^2P_{k+1,k+1} = {}^2\delta_k \times \frac{P_{k+1,k}}{1 - 2\rho_k} - {}^2\delta_k \times \frac{P_{k+1,k}}{1 - 2\rho_k} \times C_{k+1}^T {}^2W_k^{-1} C_{k+1} \times \frac{P_{k+1,k}}{1 - 2\rho_k}, \tag{9}$$

where

$$\begin{aligned}
C_{k+1} &= \frac{\partial h(x_k)}{\partial x} \Big|_{x_k = \hat{x}_{k+1,k}}, \quad {}^2W_k = \frac{C_{k+1}P_{k+1,k}C_{k+1}^T}{1 - {}^2\rho_k} + \frac{{}^2\hat{R}_{T,k+1}}{{}^2\rho_k}, \\
{}^2K_k &= \frac{P_{k+1,k}}{1 - {}^2\rho_k} C_{k+1}^T {}^2W_k^{-1}, \\
{}^2\delta_k &= 1 - [{}^2y_{k+1} - h(\hat{x}_{k+1,k})]^T \times {}^2W_k^{-1} \times [{}^2y_{k+1} - h(\hat{x}_{k+1,k})], \\
{}^2\hat{R}_{T,k+1} &= \frac{{}^2\bar{R}_{T,k+1}}{1 - {}^2\beta_R} + \frac{R_{k+1}}{{}^2\beta_R}, \quad {}^2\beta_R = \frac{\sqrt{\text{tr}(R_{T,k+1})}}{\sqrt{\text{tr}({}^2\bar{R}_{T,k+1})} + \sqrt{\text{tr}(R_{T,k+1})}}, \\
{}^2\rho_k &= \frac{\sqrt{\text{tr}({}^2\hat{R}_{T,k+1})}}{\sqrt{\text{tr}(C_{k+1}P_{k+1,k}C_{k+1}^T) + \sqrt{\text{tr}({}^2\hat{R}_{T,k+1})}}}.
\end{aligned}$$

Step 3.2. Fusion of the main circuit system observational data

$$\hat{x}_{k+1,k+1} = {}^2\hat{x}_{k+1,k+1} + {}^1K_k ({}^1y_{k+1} - h({}^1\hat{x}_{k+1,k+1})), \quad (10)$$

$$P_{k+1,k+1} = {}^1\delta_k \frac{{}^2P_{k+1,k+1}}{1 - {}^1\rho_k} - {}^1\delta_k \frac{{}^2P_{k+1,k+1}}{1 - {}^1\rho_k} \times {}^2C_{k+1}^T \times \frac{{}^2P_{k+1,k+1}}{1 - {}^1\rho_k}. \quad (11)$$

Here,

$$\begin{aligned}
{}^2C_{k+1} &= \frac{\partial h(x_k)}{\partial x} \Big|_{x_k = {}^2\hat{x}_{k+1,k+1}}, \quad {}^1W_k = {}^2C_{k+1} \left( \frac{{}^2P_{k+1,k+1}}{1 - {}^1\rho_k} \right) {}^2C_{k+1}^T + \frac{{}^1\hat{R}_{T,k+1}}{{}^1\rho_k}, \\
{}^1K_k &= \frac{{}^2P_{k+1,k+1}}{1 - {}^1\rho_k} \times {}^2C_{k+1}^T \times {}^1W_k^{-1}, \\
{}^1\delta_k &= 1 - [{}^1y_{k+1} - h({}^2\hat{x}_{k+1,k+1})]^T \times {}^1W_k^{-1} \times [{}^1y_{k+1} - h({}^2\hat{x}_{k+1,k+1})], \\
{}^1\hat{R}_{T,k+1} &= \frac{{}^1\hat{R}_{T,k+1}}{1 - {}^1\beta_R} + \frac{R_{T,k+1}}{{}^1\beta_R}, \quad {}^1\beta_R = \frac{\sqrt{\text{tr}(R_{T,k+1})}}{\sqrt{\text{tr}({}^1\bar{R}_{T,k+1})} + \sqrt{\text{tr}(R_{T,k+1})}}, \\
{}^1\rho_k &= \frac{\sqrt{\text{tr}({}^1\hat{R}_{T,k+1})}}{\sqrt{\text{tr}({}^2C_{k+1}{}^2P_{k+1,k+1}{}^2C_{k+1}^T) + \sqrt{\text{tr}({}^1\hat{R}_{T,k+1})}}}.
\end{aligned}$$

From this the location set  $E(\hat{x}_{k+1,k+1})$ ,  $P_{k+1,k+1}$  of the targets can be estimated.

After the initialization of the main circuit system is completed, the prediction set of the target location is calculated by the application of equations (6) and (7), which are exactly the same as the prediction process of the single machine. It can also be

seen from equations (6) and (7) that the results of the prediction are related only to the state of the target at the previous time, while irrelevant to the state of the circuit system, that is, no matter what the state of the two circuit systems is, the prediction set of the target is the same, so it is only required for the prediction step to complete the main circuit system. Then the main circuit system calculates the corresponding observation set according to the observational data of the collaborative circuit system and its own observational data, and calculates the intersection of the three sets by the application of the RSE update algorithm twice, and then the fusion result of the observational data can be obtained. The aforementioned process to solve the intersection of these three sets of processes can be divided into the following two steps:

1) First solve the intersection of the observation sets  $S_y^2$  of the observation set  $E(\hat{x}_{k+1,k})$ ,  $P_{k+1,k}$  and the collaborative circuit system. This step is exactly the same as the RSE of the single machine, therefore, the calculation result  $E({}^2\hat{x}_{k+1,k+1})$ ,  ${}^2P_{k+1,k+1}$  is an ellipsoidal set that meets the equation (6), denoted as  $E_{\text{temp}}$ ;

2) Solve the intersection of  $E_{\text{temp}}$  and the main circuit system observation set. In this process,  $E_{\text{temp}}$  is regarded as a prediction set, introduce the set into the RSE method, and make use of the main circuit system to update its observation set so as to obtain the intersection of these two sets, which is the time observational data fusion result.

Under normal circumstances, according to the assumption of RSE, both of the circuit systems of the observational data set contain the real system state points; therefore, the intersection must be non-empty. However, when some extreme conditions are encountered (for example: Some observers fail and result in large errors in the observational data), the observation sets of the two circuit systems may not intersect, that is  $E_{\text{temp}} \cap S_y^1 = \phi$ . In this case, in order to be able enable the algorithm to be carried on recursively, we can directly take the estimated results in 1) as the final observation results, and carry out the next recursive operation.

### 3. Simulation experiment

The simulation experiment is carried out by Matlab on the PC platform. And the experimental parameters are as the following: Envelope matrix of the process noise:  $Q = \text{diag}\{0.0001, 0.0001, 0.0001\}$ ; Envelope matrix of the observation noise:  $R = \text{diag}\{0.001, 0.001, 0.001\}$ ; Envelope matrix of the initial target state:  $P_0 = \text{diag}\{0.1, 0.1, 0.1\}$ ; Weight value:  $\omega_1 = 0.01$ ,  $\omega_2 = 0.0005$ ,  $\omega_3 = 2.7$ ,  $\omega_4 = 1$ ,  $\omega_5 = 1.1$ .

Figure 3 shows the trajectory of the active observation process of the two circuits. The top left figure is the aerial view of the grid. The other three figures are the projection of the trajectory of the circuit in plane  $x - y$ , plane  $y - z$  and plane  $x - z$ . For the ease of viewing, all the ellipsoids in the figure are all magnified by a factor of 15. The middle solid line in the figure indicates the trajectory of the movement of the target, and the other two solid lines represent the trajectory of the two circuit systems, respectively. It can be seen from the figure that, with the progress of observation, the uncertain ellipsoid set of the target state becomes

smaller and smaller, that is, the observation on the target becomes more and more precise. To further validate the method proposed in this paper, we assume that there is a sudden change in the trajectory of the target, but it can be clearly seen from the figure that this mutation does not affect the tracking of the target by the two circuit systems, and that they can still properly plan their own trajectories and perform observation on the target, the ellipsoid set of the target state can converge to a relatively smaller stable value.

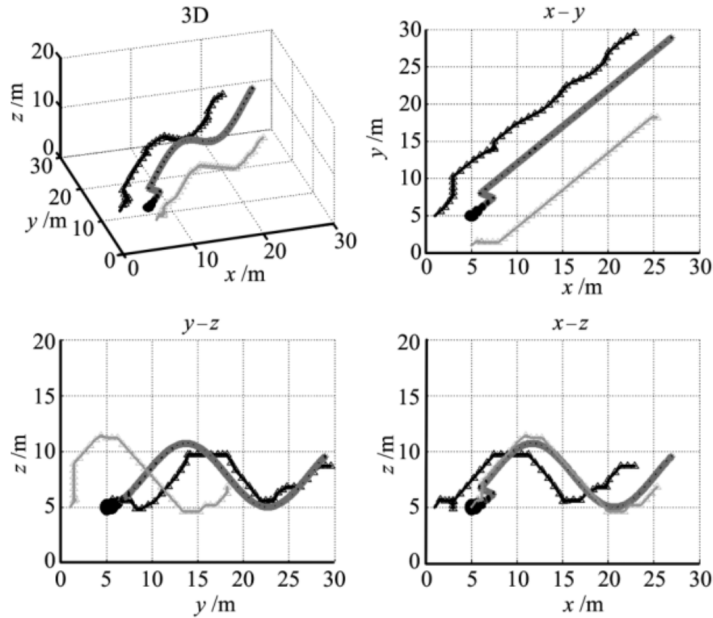


Fig. 3. Trajectory of the active observational data fusion process

The length of three axes of the ellipsoid can reflect the size of the ellipsoid, so the trajectory of the envelope matrix  $P_{k,k}$  in equation (8) can be used as a parameter to measure the ellipsoid size. Figure 4 shows the variation trend of the trajectory of the matrix  $P_{k,k}$ , which is a quantitative description of the variation of the ellipsoid set. The solid line in the figure shows the results obtained with the observational data of only one circuit system, and the dotted line shows the result of data fusion by the adoption of two circuit systems. The results of the observational data fusion are significantly superior to those of the single machine observations (0.0090 vs. 0.0034) from both the convergence rate and the observation precision. In the figure, there is a sudden variation in the vicinity of 5s, which is due to the sudden variation of the target trajectory. However, after the mutation, the two circuits system can make the observational ellipsoid converge to a relatively smaller value by properly planning the respective trajectories, which indicates that the algorithm proposed in this paper has good stability.

Figure 5 shows the variation of the observational data fusion observational angle



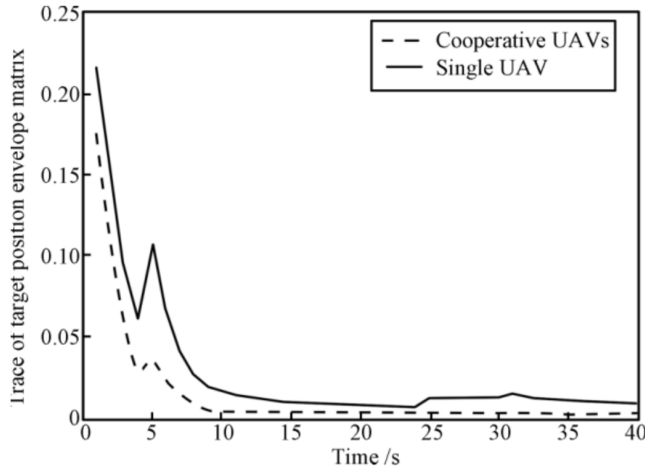


Fig. 4. Variation of the trajectory of target state envelope matrix

during the process of observation, and the observational data fusion angle maintains at  $90^\circ$  or so in the whole observation process. Therefore, the method proposed in this paper can effectively plan the path of the circuit system, and realize the approximation optimization fusion of the observational data on the target.

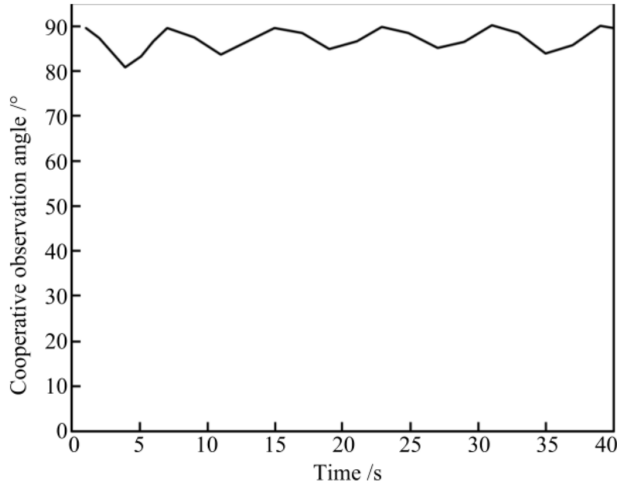


Fig. 5. Variation of the observational data fusion angle

In order to illustrate the fastness of the proposed method in more detail, we compare the observational data fusion method mentioned in Section 2.2 with the method proposed in literature [9]. Table 1 lists the computation time of the two algorithms under the scenario of three circuit system observational data fusion. It can be clearly observed from the table that, the average time of iteration is 0.313 s

at each step of the data fusion method proposed in this paper, which is very close to that of the single-circuit system observation method. From the simple passive observational data fusion method, the data fusion process of the method proposed in this paper does not introduce too many computations. While the method proposed in literature [10] consumes an average of 0.391 s for the iteration each time, which is 1.2 times of the method proposed in this paper. Therefore, it can be seen that the algorithm proposed in this paper is more applicable for the real-time application environment. And the differences between these two algorithms are shown in literature [11].

Table 1. Time comparison of the two observational data fusion methods

Algorithm	Time (s)
Single circuit system observation method	0.311
Three-circuit system observational data fusion (the method proposed in this paper)	0.313
Three-circuit system observational data fusion (the method put forward in literature [7])	0.391

## 4. Conclusion

In this paper, a RSE-based multi-circuit data active observation fusion method is put forward. This method makes use of the calculation characteristics of RSE and integrates the fusion process of the observation results of the multi-circuit system into the estimation algorithm, so as to realize the real-time optimization observation of the target. The main advantages of this method are summarized as the following:

- 1) The optimization observation on the target can be realized for the circuit system so as to ensure the precision of the observation.
- 2) On the basis of the estimation method, the observational data fusion of the two circuits is implemented through the introduction of less calculation to improve the rapidity of the algorithm.
- 3) In addition, less approximation process is introduced to improve the precision of the observation results. Finally, the simulation results are provided to verify the feasibility and effectiveness of the algorithm.

## References

- [1] C. L. FU, D. L. GONG, L. I. JIE: *Multi-sensor consistency data fusion algorithm in observation uncertainty*. *Transducer & Microsystem Technologies* 32 (2013), No. 7, paper 113.
- [2] U. ZENGİN, A. DOĞAN: *Real-time target tracking for autonomous uavs in adversarial environments, a gradient search algorithm*. *IEEE Transactions on Robotics* 23 (2007), No. 2, 294–307.
- [3] K. ZHOU, S. I. ROUMELIOTIS: *Optimal motion strategies for range-only constrained*

- multisensor target tracking*. IEEE Transactions on Robotics *24* (2008), No. 5, 1168–1185.
- [4] G. GU, P. R. CHANDLER, C. J. SCHUMACHER, A. SPARKS, M. PACHTER: *Optimal cooperative sensing using a team of UAVs*. IEEE Transactions on Aerospace and Electronic Systems *42* (2006), No. 4, 1446–1458.
  - [5] L. WANG, J. W. WAN, Y. H. LIU, J. X. SHAO: *Cooperative localization method for multi-robot based on PF-EKF*. Science in China Series F: Information Sciences *51* (2008), No. 8, 1125–1137.
  - [6] B. ZHOU, J. D. HAN: *A UD factorization-based adaptive extended set-membership filter*. Acta Automatica Sinica *34* (2008), No. 2, 150–158.
  - [7] J. OUSINGSAWAT, M. E. CAMPBELL: *On-line estimation and path planning for multiple vehicles in an uncertain environment*. International Journal of Robust and Nonlinear Control *14* (2004), No. 8, 741–766.
  - [8] P. YANG, R. A. FREEMAN, K. M. LYNCH: *Multi-agent coordination by decentralized estimation and control*. IEEE Transactions on Automatic Control *53* (2008), No. 11, 2480–2496.
  - [9] C. D. PATHIRANAGE, K. WATANABE, K. IZUMI: *T-S fuzzy model adopted SLAM algorithm with linear programming based data association for mobile robots*. Soft Computing *14* (2010), No. 4, 345–364.
  - [10] D. ZU, J. D. HAN, D. L. TAN: *Transverse vibrations of non-homogeneous rectangular plates with variable thickness*. Acta Automatica Sinica *33* (2007), No. 10, 1036–1042.
  - [11] E. A. ARKENBOUT, J. C. F. DE WINTER, P. BREEDVELD: *Robust hand motion tracking through data fusion of 5DT data glove and nimble VR kinect camera measurements*. Sensors (Basel) *15* (2015), No. 12, 31644–31671.

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